REFUTING A FRACTION MISCONCEPTION: A BRIEF INTERVENTION PROMOTES TEACHERS' CONCEPTUAL CHANGE

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Research shows that some teachers overgeneralize the whole number rule of "multiplication always makes bigger" to multiplication with fractions. The present study explores this misconception not only with fractions with unfamiliar denominators (e.g., thirty-fifths) but also fractions with familiar denominators (e.g., halves) and evaluates an intervention for remediating the misconception. In- and pre-service teachers (N = 100) completed a direction of effects task prior to and after random assignment to either read a control text or a refutation text that directly refuted the misconception. The misunderstanding was present among items with unfamiliar and familiar denominators, demonstrating the pervasiveness of the misconception. Overall, the intervention was found to be effective, showing promise of refutation texts for promoting teachers' mathematics conceptual change.

Keywords: Rational Numbers, Teacher Knowledge, Number Concepts and Operations, Elementary School Education

Studies of teachers' mathematics knowledge have shown that many possess a limited conceptual and procedural understanding of fractions (e.g., Ma, 1999; Moseley, Okamoto, & Ishida, 2007), limiting their preparedness to support their students' fraction learning. Preservice teachers struggle to comprehend the most foundational conceptual understanding of fraction arithmetic, which is the *direction of effects* that an operation produces (e.g., whether multiplying by a given value makes a number smaller or larger; Siegler & Lortie-Forgues, 2015).

Research on direction of effects errors has concentrated on multiplication of fractions (Siegler & Lortie-Forgues, 2015). A common misconception that individuals hold is to overgeneralize the whole number rule that "multiplication always makes bigger" to multiplication with fractions, when in fact, multiplying a positive number by a fraction that is less than one does *not* make that positive number bigger. In a study conducted by Siegler and Lortie-Forgues (2015), 41 preservice teachers completed a direction of effects task for which they were asked to predict without calculating whether the answer to an inequality would be larger or smaller than the larger fraction in the multiplication problem. The fractions used in the task were selected to be too large and uncommon so that the items could not be solved easily with mental arithmetic (e.g., "True or False: 41/66 x 19/35 > 41/66"; Siegler & Lortie-Forgues, 2015). The preservice teachers answered correctly less often than chance on multiplication items designed intentionally to capture the misconception (i.e., multiplication items with fractions between zero and one).

The present study adds to the literature in two important ways by (a) examining the presence of the direction of effects misconception for items involving unfamiliar denominators and items involving familiar denominators and (b) evaluating a brief intervention for remediating the misconception.

Theoretical Framework

Individuals who transfer the whole number rule that "multiplication makes bigger" to all multiplication problems with fractions may need to undergo conceptual change to overcome this

misconception. Conceptual change is a process in which individuals restructure conceptual knowledge and shift misconceptions prompted by discrepant information (Carey, 2009). We draw from the Cognitive Reconstruction of Knowledge Model (CRKM) model of conceptual change that discerns between characteristics of the learner (i.e., their prior conceptions and motivation) and of the message (i.e., whether the learning material is comprehensible, coherent, compelling, and plausible; Dole & Sinatra, 1998). This model is particularly useful for our research because it explains why individuals might shift their misconceptions about fraction multiplication based on novel information presented in a refutation text.

Refutation texts have been shown to be an effective tool for promoting conceptual change in science education (e.g., Broughton, Sinatra, & Nussbaum, 2012; Tippet, 2010). These texts are designed to state a misconception, refute those ideas, and then present the accurate explanations as plausible and fruitful alternatives (Hynd, 2001). Whereas positive effects of refutation texts for the remediation of misconceptions have been demonstrated in science education for years (e.g., Diakidoy, Mouskounti, & Ioannides, 2011), the texts have not yet been explored extensively for addressing mathematics misconceptions. A noted exception is research conducted by Lem and colleagues (Lem, Onghena, Vershaffel, & Van Dooren, 2017) that explored the use of refutation texts for correcting students' misconceptions about box-and-whisker plots.

Present Study

To our knowledge, there are currently no studies investigating the presence of the "multiplication always makes bigger" misconception when fractions being multiplied have common denominators (e.g., halves, fourths) and no work investigating an instructional intervention for remediating this specific misconception. In response, the present study was designed with the following two objectives: (a) to assess if pre- and in-service teachers correctly answer less often than chance on easy, medium and hard direction of effects problems involving multiplication of fractions between zero and one and (b) to evaluate the effectiveness of a refutation text that targets the misconception for promoting conceptual change.

Method

Participants

Participants were pre-service and in-service elementary school teachers (N = 100), 38% self-reported as male, 42% White, 45% Asian American/Pacific Islander, 1% Hispanic, 3% Black/African American, 8% Mixed and 5% Other. The mean age was reported as 21.1 years (SD = 3.7). All participants were recruited via a Qualtrics survey panel that exclusively targeted in- and pre-service elementary teachers.

Measures

Texts. Two texts were created, a refutation text and a control text. The refutation text consisted of 364 words, had an 11th grade Flesch-Kincaid reading level, and stated and directly refuted the "multiplication always makes bigger" misconception. For example, an excerpt reads: "When multiplying numbers, you may think that the product will always be greater than the original number. This is incorrect! It is only when you multiply a positive number by a number greater than 1 that the product will be greater than the original number." This direct refutation was followed by a description of when multiplication makes bigger, and does not. The control text was a 100-word expository text paired with an illustration that was adapted from Hake (2007) that describes a strategy for multiplying fractions: "We often translate the word *of* into a multiplication symbol. We find 1/2 of 1/2 by multiplying: 1/2 of 1/2 becomes 1/2 × 1/2 = 1/4."

Direction of effects task. The direction of effects task assessed participants' understanding of the direction of effects of arithmetic, which was used to measure participants' conceptual understanding of fraction addition and multiplication (Siegler & Lortie-Forgues, 2015). Participants were asked to evaluate the accuracy of addition inequalities in the form of $a/b + c/d > c/d$ and multiplication inequalities in the form of $a/b \times c/d > c/d$. Participants answered four addition inequalities and four multiplication inequalities within three categories of varying difficulty, resulting in 24 total items. The "easy" items all included the fraction 1/2 paired with a fraction with a familiar denominator such as fourths (e.g., 3/4 × 1/2 > 1/2); "medium" items included familiar denominators but did not present any unit fraction such as 1/2 (e.g., 9/10 × 2/5 > 2/5); and "hard" items included the items used in a previous study (Siegler & Lortie-Forgues, 2015) with unfamiliar denominators (e.g., 19/35 × 41/66 > 41/66). For each inequality, participants were instructed to decide, without calculating, whether the answer would be greater than the answer indicated in the inequality.

Procedure

All materials were presented via an online Qualtrics survey. After agreeing to participate, teachers completed the pretest direction of effects task. Then, teachers were randomly assigned to either the refutation text or the control text group. After reading the text, teachers were asked to complete the direction of effects task again which served as a posttest assessment. The total survey required an average of 18 minutes (*SD* = 19).

Results

We conducted a simple one sided test of proportions to determine if teachers correctly answered multiplication problems with fractions between zero and one less than half of the time. Analysis of teachers' pretest performance on the easy and medium items with familiar denominators and hard items with unfamiliar denominators revealed that teachers chose the correct item about 49% of the time, which was not significantly different than chance (50%; chi-squared = .375, $p = .27$). When considering all 24 pretest items, 31% of teachers were correct on 100% of the 18 addition and multiplication items for which the greater-than inequality was true and 0% on the 6 multiplication items for which the greater than inequality statement was false. In other words, 31% of teachers at pretest showed the true misconception that "multiplication always makes bigger."

Of the 57 participants in the refutation text group, 9 participants continued to show the true misconception (16%) at posttest. We then looked more closely at participants' performance on the six multiplication items that were designed to capture the misconception. We focused attention on the refutation group participants who answered at least one of the misconception items incorrectly at posttest ($n = 28$). Of this subgroup, participants correctly answered at a rate similar to chance for five of the six items ($p > .05$) and less often than chance on one of the six items with unfamiliar denominators: 19/35 × 41/66 > 41/66 (71.4%; $p = .01$).

To assess the effectiveness of the refutation text intervention, we ran a repeated measures ANOVA with Arithmetic Operation (addition or multiplication) as a within-subject factor and text condition (refutation text or control) as between-subject factor and number of correct judgments as the dependent variable yielded significant main-effects of Arithmetic Operation ($F(1,98) = 64$, $p < .001$, *partial-eta squared* = .395) and main effects of condition that are approaching significance ($F(1,98) = 2.24$, $p = 0.14$, *partial-eta squared* = 0.022). These were qualified by a significant Operation x Condition interaction ($F(1,98) = 4.462$, $p < .001$, *partial-eta-squared* = 0.043). As expected, post hoc comparisons with Bonferroni correction showed no significant differences between ref-text and control groups in addition knowledge (89% vs 89%,

$t(42) = 0.03$, $p = .98$, *Cohen's d* = 0.00). However, we found significant differences between refutation text and control groups in multiplication knowledge (74% vs 63%, $t(42) = 2.45$, $p = .02$, Hedge's $g = 0.56$).

Discussion

We assessed the effectiveness of a refutation text intervention for the remediation of a misconception regarding fraction arithmetic. Though there is some research on the use of refutation texts to shift students' misconceptions about mathematical topics (Lem et al., 2017), our study investigates the use of a refutation text to shift teachers' misconceptions about fraction multiplication. We also explore whether the misconception is specific to multiplication of unfamiliar fractions or common fractions more generally. Overall, we found that teachers answered easy, medium and hard fraction multiplication problems correctly at about the same rate as guessing, suggesting that the misconception is even more widespread and problematic than demonstrated in prior research (Siegler & Lortie-Forgues, 2015). Furthermore, we found that teachers who read a refutation text had fewer misconceptions about fraction multiplication compared with those who read a control text. That is, refutation texts have the potential to substantially impact teachers' mathematics misconceptions. This finding has important implications for research on conceptual change in mathematics and refutation texts.

Teachers Hold the Multiplication Misconception

Our results showed that a sample of pre- and in-service teachers answered fraction multiplication problems with fractions between zero and one at about the same rate as by chance. This finding is consistent with prior research showing that pre-service teachers were not successful in answering items regarding directions of effects in multiplication at a rate better than chance (Siegler & Lortie-Forgues, 2015). This replication of the finding suggests that many pre- and in-service teachers are in need of remediating misconceptions that may be limiting their readiness to advance their students' fraction understanding.

Refutation Texts Support Mathematics Conceptual Change

Teachers who were randomly assigned to read a refutation text that directly refuted the "multiplication always makes bigger" misconception had fewer misunderstandings at posttest than those assigned to read a control text, with modest effect sizes (Hedge's $g = 0.56$). Refutation texts targeting specific misconceptions thus hold potential as a relatively quick method for promoting teachers' conceptual change.

Future Directions

Although the present study shows promise of a refutation text serving as an instructional approach for targeting a specific fraction misconception, replication studies are required before widespread recommendation of refutation texts for supporting mathematics conceptual change. In light of the limited studies thus far exploring refutation texts for mathematics learning, there are multiple research curiosities that future work can explore, such as refutation texts for remediating not only other fraction misconceptions but also misconceptions in other mathematics content areas. Researchers should also explore the effectiveness of refutation interventions among not only teachers but also students, as such texts may hold promise for professional development opportunities and classroom instruction alike.

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